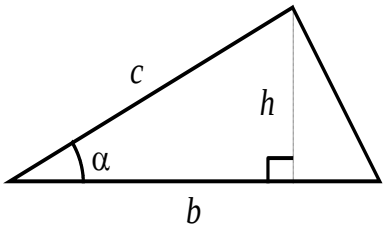
Area of a Triangle

The formula $A = \frac{1}{2}bh$ gives the area of a triangle.



In the diagram at the right, $\sin\alpha = \frac{h}{c}$, so $h = c\sin\alpha$. Using substitution, we derive the formula $A = \frac{1}{2}bc\sin\alpha$.

Depending on which angles and sides are known, the formulas $A = \frac{1}{2}ac\sin\beta$ and $A = \frac{1}{2}ab\sin\gamma$ can also be used.

Using the law of cosines, it is possible to derive a formula for the area of a triangle that involves only the lengths of the sides of the triangle. The formula is known as "Heron's Formula" after Heron of Alexandria, who is believed to have discovered it around AD 75.

Heron's Formula: The area of a triangle with sides of lengths *a*, *b*, and *c* is given by:

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \text{ where } S = (a+b+c)/2.$$

Examples:

Find the area of the triangle with $\alpha = 39.4^\circ$, $b = 12.6$, and $c = 13.7$

Find the area of a triangle with $\alpha = 56.3^\circ$, $\beta = 41.2^\circ$, and $a = 9.8$

Find the area of the triangle with $a = 12$, $b = 8$, and $c = 6$

Find the area of a triangle with $a = 346$, $b = 234$, and $c = 422$